Reservation Prices

A *reservation price* is the monetary amount a consumer is willing to give up to acquire an extra marginal unit of some good. Reservation prices are also referred to as the customer's *willingness to pay.* Formally, if x^* denotes the optimal solution to (E.1), the reservation price, denoted v_i, for an additional unit of good i is given by

$$v_i \equiv \frac{\partial \tilde{u}(\mathbf{x}^*)}{\partial x_i}, \tag{E.3}$$

where $\tilde{u}(\mathbf{x}^*)$ is the monetary utility (E.2). The first-order conditions of the budget problem imply $\frac{\partial \tilde{u}(\mathbf{x}^*)}{\partial x_i} = p_i$ since $\tilde{u}'(w) = \pi = 1$ when utilities are measured in dollars. Combining this with (E.3) implies that $v_i = p_i$. Thus, a customer's reservation price for goods that are *currently consumed* is simply the current market price. The reason for this equivalence, intuitively, is that if our customer valued another unit of good i at strictly more than its market price, then he would be able to increase his utility by reducing consumption of other goods and increasing his consumption of good i. Since our customer is assumed to be maximizing utility, this cannot occur.

On the other hand, for goods i that are not being consumed, so $x_i^* = 0$, the first-order conditions to (E.1) imply $\frac{\partial \tilde{u}(\mathbf{x}^*)}{\partial x_i} < p_i$, or equivalently $v_i < p_i$. In other words, by (E.3) the customer's reservation price for the first unit of good i is strictly less than its current market price. Moreover, the customer would change only his allocation and buy good i if its price p_i dropped below his reservation price v_i.

This formal analysis of reservation price is arguably less important in practice than the informal concept—namely, that the reservation price is the maximum amount a customer is willing to pay for an additional unit of good i. And to entice a customer to buy good i, the price must drop below his reservation price. Still, the analysis highlights the important fact that reservation prices are not "absolute" quantities. Like utility for money, they depend on customers' preferences, wealth, their current consumption levels, and the prices of other goods the customers may buy; change one of these factors, and customers' reservation price may change.

Lotteries and Stochastic Outcomes

Many choices in life involve uncertain outcomes, such as buying insurance, making investments or eating at a new restaurant. How do customers respond to these sorts of uncertainties? The theory of choice under uncertainty is a deep and extensive topic. Here, we outline the basic ideas and highlight the main concepts.

Consider again a discrete, finite set of n alternatives, $X = \{x_1, \dots, x_n\}$. Let $\mathcal{P}$ be the class of all probability distributions $P(\cdot)$ defined on X. That is, $P \in \mathcal{P}$ is a function satisfying $\sum_i P(x_i) = 1$ and $P(x_i) \geq 0$ for $i = 1, \dots, n$. One can think of each P as a "lottery," the outcome of which is that the customer is left with one of the alternatives x_i according to the distribution P.

What can we say about the customer's preference for these various lotteries? Specifically, when can we say that for any two lotteries P_1 and P_2, customers "prefer" one over the other (denoted by $P_1 \succ P_2$)?

To answer this question we again need to make some assumptions on customer preferences. First, we will assume there exists a preference relation $\succ$ on the n different outcomes x_i as before. Second, for any two lotteries P_1 and P_2, consider a compound lottery parameterized by α as follows: